Solutions to random exercises from Chapter 4

4.1

1.

x	y	$\frac{\Delta y}{\Delta x}$
-8	80	$\frac{-17}{1}$
-7	63	$\frac{-15}{1}$
-6	48	$\frac{-13}{1}$
-5	35	$\frac{-11}{1}$
-4	24	$\frac{-9}{1}$
-3	15	$\frac{-7}{1}$
-2	8	$\frac{-5}{1}$
-1	3	

Variable rate of change because for equal changes in x, there are different corresponding changes in y.

2.

x	y	$\frac{\Delta y}{\Delta x}$
0	1	$\frac{4}{2}$
2	5	$\frac{1}{2}$
4	6	$\frac{6}{2}$
6	12	$\frac{4}{2}$
8	16	$\frac{3}{2}$
10	19	$\frac{1}{2}$

Variable rate of change because for equal changes in x, there are different corresponding changes in y.

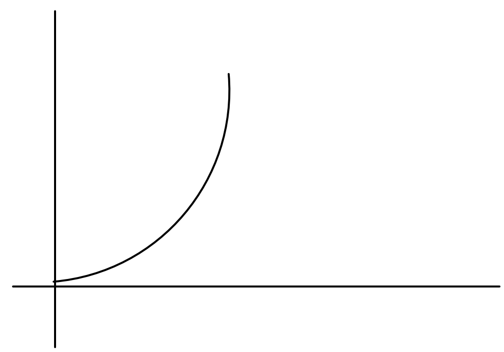
12	20	$\frac{5}{2}$
14	25	$\frac{4}{2}$
16	29	

3. This function does not have a constant rate of change because the curvature of the graph demonstrates that as x changes in equal amounts the change in y varies.

4. This function does have a constant rate of change because for equal changes in *x* there are equal changes in *y*. Specifically for every change of 1 in *x* there is a change of 2 in *y*.

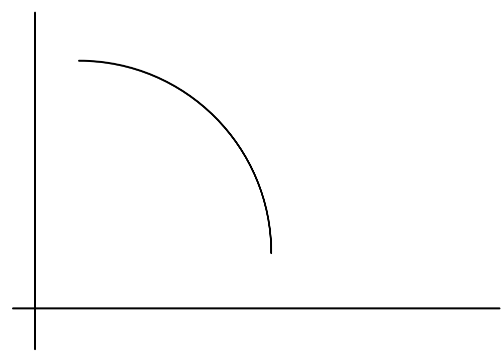
5. (Answers vary)

x	*y*
0	0
1	1
2	4
3	9
4	16
5	25



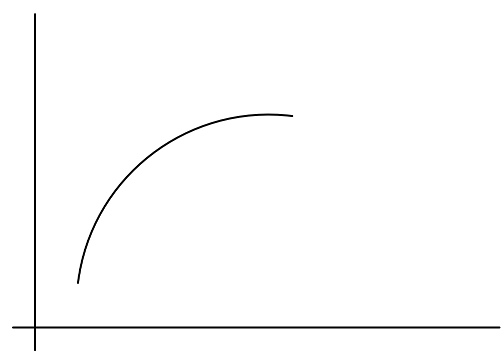
6. (Answers vary)

x	*y*
1	50
2	48
3	44
4	38
5	30
6	20



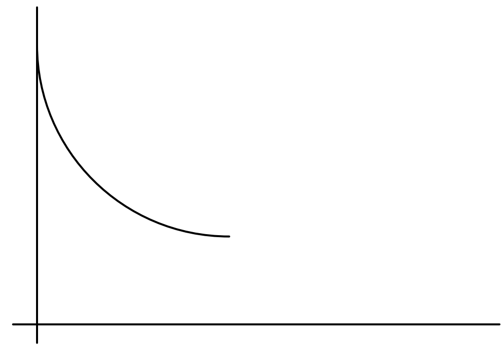
7. (Answers vary)

x	*y*
1	1
4	2
16	4
25	5
36	6



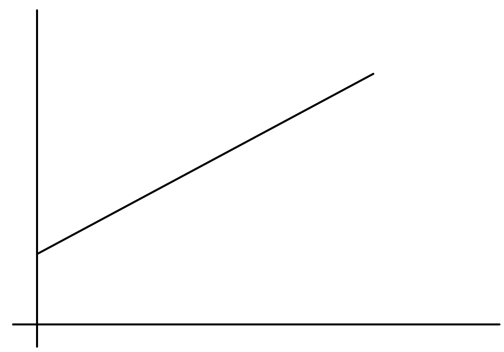
8. (Answers vary)

x	*y*
0	18
1	11
2	6
3	2
4	2



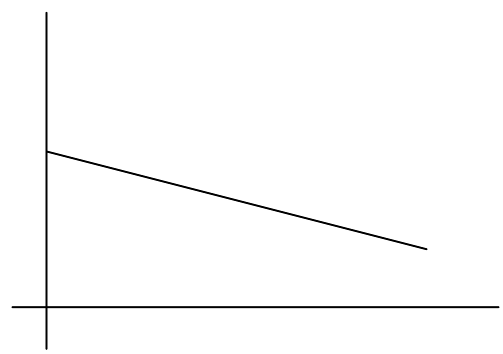
9. (Answers vary)

x	*y*
0	5
1	7
2	9
3	11
4	13



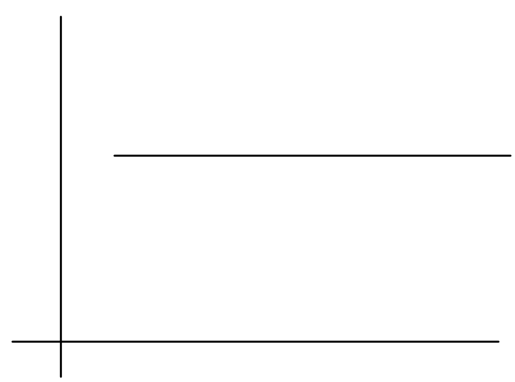
10. (Answers vary)

x	*y*
10	34
11	31
12	28
13	25
14	22
15	19



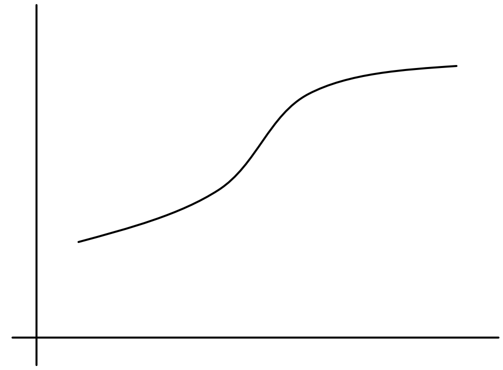
11. (Answers vary)

x	*y*
3	12
4	12
5	12
6	12
7	12
8	12



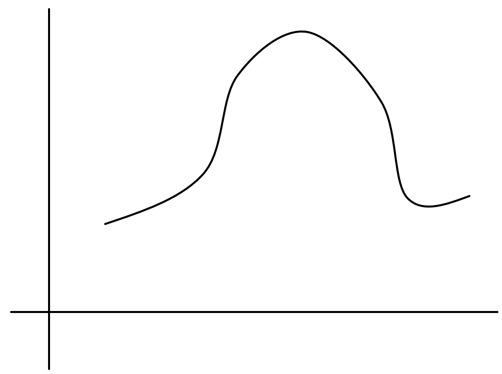
12. (Answers vary)

x	*y*
3	5
4	8
5	13
6	16
7	18
8	19



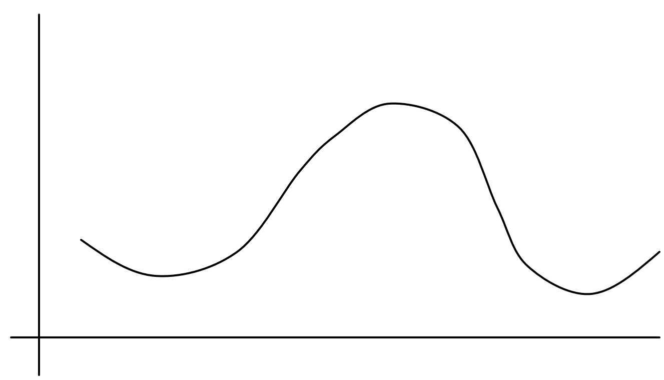
13. (Answers vary)

x	*y*
2	4
3	5
4	8
5	13
6	15
7	16
8	15
9	12
10	7
11	4
12	3
13	4
14	8



14. (Answers vary)

x	*y*
2	4
3	5
4	8
5	13
6	15
7	16
8	15
9	12
10	7
11	4
12	3
13	4
14	8



15. (a) and *h(x)*
(b) and *f(x)*
(c) and *g(x)*
(d) does not fit any of the three functions

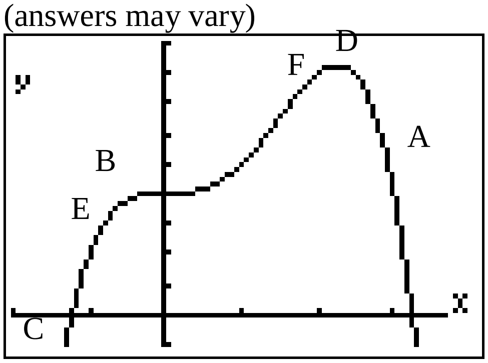
16. $f(x)$ = D; $g(x)$ = B; $h(x)$ = A

17. $f(x)$ and $g(x)$ both have variable rates of change, $h(x)$ has a constant rate of change.

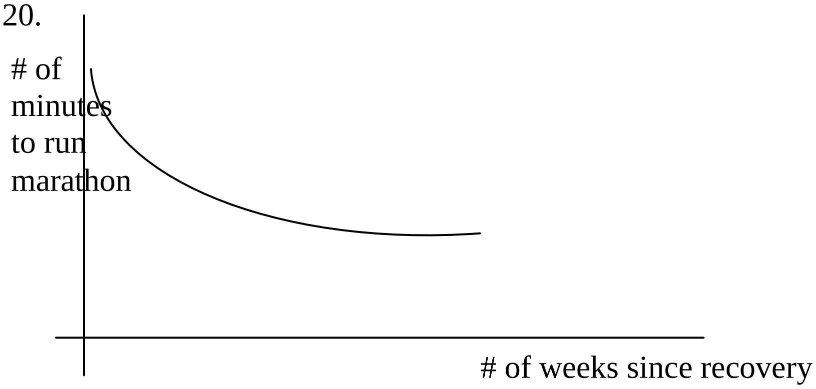
18.

x	y	Δy	$\Delta(\Delta y)$
-3	-33	17	
-2	-16	11	-6
-1	-5	5	-6
0	0	-1	-6
1	-1	-7	-6
2	-8	-13	-6
3	-21		

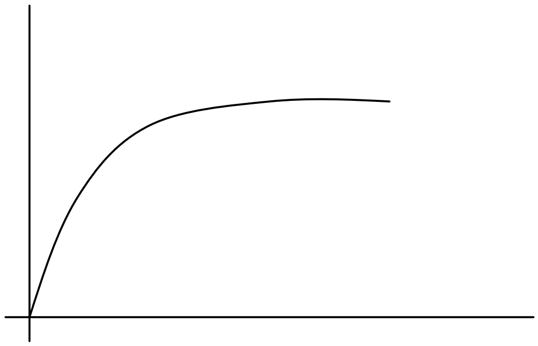
19. (answers may vary)



20.



21.



22. $\dfrac{-2-0}{3--2}=\dfrac{-2}{5}$

23. $\dfrac{-1-3}{2--1}=\dfrac{-4}{3}$

25. Since the intervals for the change in x in the table are one unit apart the best we can estimate the instantaneous rate of change at x = 0 is $\dfrac{4-3}{0--1}=\dfrac{1}{-1}=-1$. (You could also use x =1 and x = 0 to estimate.)

26. As x increases from x = 0 to about x = 5 the function f is increasing, concave up. As x increases from x = 5 to x = 15 the function f is increasing, concave down. (Note that point **P** is a point of inflection because the concavity changes at that point.)

27. a. The function F(s) is increasing approximately for posted speed limits from 20 to 57 miles per hour and it is decreasing approximately for posted speed limits from 57 to 70 miles per hour.

b. F(s) is primarilyconcave down. This tells us that as the posted speed lmit increases from 20 to 57 miles per hour the fatality rate increases but at a decreasing rate. After posted speed limits of 57 miles per hour the fatality rate decreases at an increasing rate.

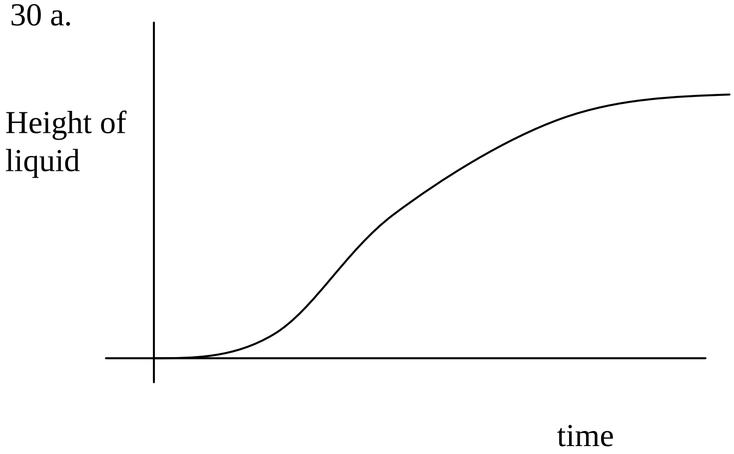
28. It is impossible to tell if the function $f(x)$ is linear or not. Simply because the graph "looks linear" is not sufficient. We do not have the scales on the axes to determine if there is a constant rate of change. Any graph can "look linear" if we look at a small enough interval of x.

29. a. The function is constant from day 22 to 24 and day 27 to 28, is increasing from day 19 to 21 and day 24 to 26, and is decreasing from day 21 to 22, day 26 to 27, and day 28 to 29.

b. Concave down because the rate of change is decreasing.

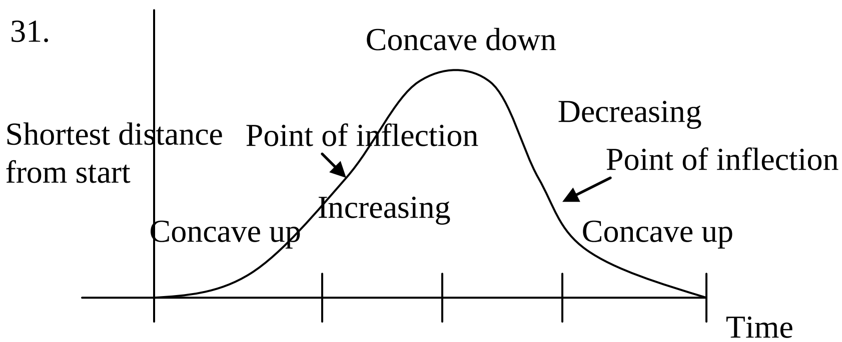
c. It tells us that the rate of temperature change is decreasing. In other words the temperature is going up but at a lesser rate.

30 a.



b. Yes, it's important to consider the water is getting poured at a constant rate because if it were not being poured at a constant rate then we could not graph height as a function of time. We must consider small intervals of time and how the height is changing in order to sketch the graph. If the rate that the liquid was poured changed then it would effect the height at different times.

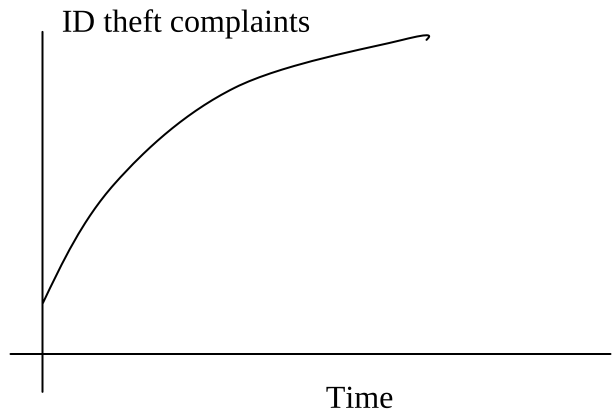
31.



32. It tells you that he/she is only looking at the change in the outputs when considering the average rate of change and not paying attention to the fact that the change in the input values is over an interval of 2 units. Therefore the change of 18 in the outputs must be divided over an interval of 2 to determine the average rate of change of 9.

33. a. This headline tells us that the number of complaints about ID theft is increasing but at a slower rate. The number of complaints are going up but not as fast.

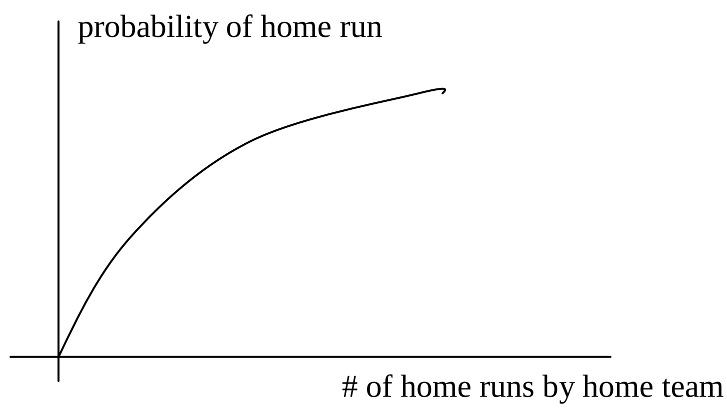
b.



c. (Answers vary)

i. The number of people moving into the state continues to rise but more slowly
ii. A population boom: People moving into town at an ever increasing rate!
iii. Ticket prices increase steadies
iv. Gasoline prices drop but have begun to level off

34. a.



b. Going into a baseball game there already exists some factors that benefit the home team such as playing on a field the players are familiar with, being at home and at a close proximity to where the game is played, and fans cheering for the home team. As the home team scores additional runs the team, fans, etc. begin to become more and more excited about the prospects of winning increasing the probability of winning. The manner in which the probability goes up could not be constant or at an increasing rate because the maximum probability of winning is 1. So it would increase less and less with each run scored, reaching a limit of winning probability at 1.

c. It could be true because as the home team scores more runs, the adrenaline of the crowd and players is contagious and it often leads to more runs. On the other hand, the visiting team without the support of

fans on their side feel more and more defeated with each additional run scored.

35. (answers vary)

Year (since 2002) d	Number of Foreclosures (F)
0	1563
1	***1115***
2	***325***
3	***100***
4	314

36. a. $\frac{4334+1832}{2}=3083$. This gives us the "middle" per capita income (in dollars) between the years 1960 to 1970 received $3,083.

b. $\frac{4334-1832}{10-0}=\frac{\$2502}{10 \text{ years}}=\frac{\$250.20}{\text{year}}$. $P(5)=1832+250.2(5)=\$3083$. If we assume that United States residents all received the same increase in their income they would have received $250.2 each year for five years resulting in an income of $3083 in 1965.

c. No, since $P(13)$ is not exactly half way between the years 10 and 20 the average per capita income would not be an accurate estimate therefore we would need to use the average rate of change.

d.

t	P	ΔP	$\Delta(\Delta P)$
0	1832		
10	4334	2502	
20	9865	5531	3029
30	18425	8560	3029
40	30013	11588	3028
50	**30013 + 14617 = 44630**	**11588 + 3029 = 14617**	**3029**

In 2010, the estimated per capita income is $44,630 in the United States.

38. a. Fertilizer initially has a positive impact on the number of bushels of apples, but eventually the fertilizer begins to impair the production of bushels and has a negative impact.

b. 200 – if no fertilizer is used, the farmer can expect 200 bushels of apples.

c. 74 – If 74 pounds of fertilizer is used, there won't by any apples being produced.

d. (answers vary) Range: $0 \le y \le 530$

e. (answers vary) Increasing from $0 \le f \le 35$

f. (answers vary) Decreasing from $35 \le f \le 74$

g. The function is concave down which means that initially even though the apple production increases with additional fertilizer, the increase in apple production becomes less and less until the increase eventually levels of and becomes a decrease in apple production with additional fertilizer applied.

h. If there was a function limit it would mean that eventually the application of more and more fertilizer would result in a constant value of apple production. Additionally, the rate of change in bushels/pound would approach 0.

40. a. $\dfrac{13285-11704}{6-0}=\dfrac{1581}{6\text{ years}}\approx\dfrac{264\text{ abortions}}{\text{year}}$. Assuming the number of abortions reported changed at a constant rate, the increase each year was 264 abortions from 1984 to 1990.

b. $\dfrac{6565-13285}{20-6}=\dfrac{-6720}{14\text{ years}}\approx\dfrac{-480\text{ abortions}}{\text{year}}$. Assuming the number of abortions reported changed at a constant rate, the decrease each year was 480 abortions from 1990 to 2004.

c. Yes, although there could possibly be other factors involved that we are not aware of, it appears that these laws did have the effect of decreasing the number of reported abortions. We can see from the table that the rate of change after 1990 becomes negative meaning that the number of reported abortions were decreasing over each two-year interval.

42. Yes. Each candidate could be telling the truth. The first candidate who stated "As long as I have been in office, the crime rate has dropped!" is speaking of how the change in the crime rate is behaving. If the crime rate has dropped it could mean that the number of crimes is increasing but at a decreasing rate (figure 1) or the number of crimes could be decreasing at a decreasing rate (fugure 2).

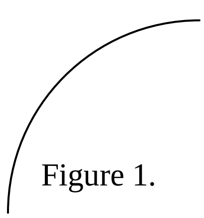


Figure 1.

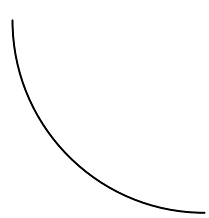


Figure 2.

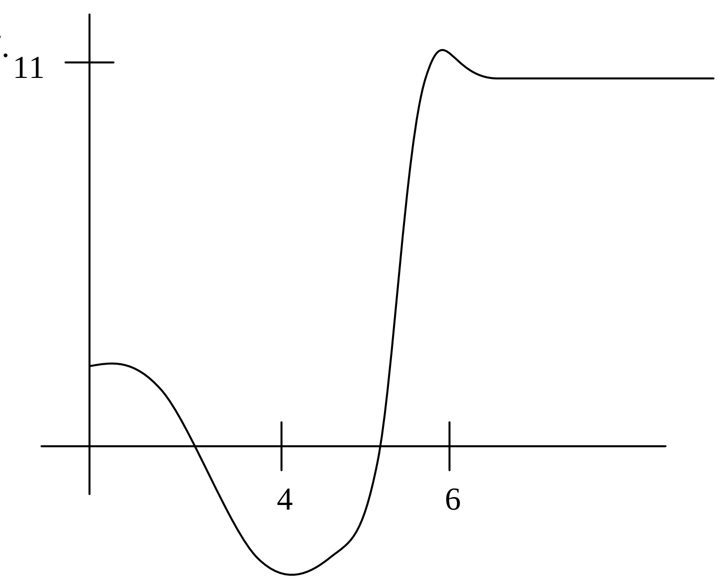
The opponent could also be telling the truth if the number of crimes are increasing such as in figure 1 above.

44. a. *N(t)* is increasing and concave up from $0 \le t \le 80$. *N(t)* is increasing and concave down from $80 \le t \le 110$.

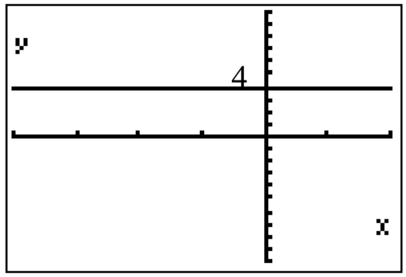
b. This means that for the 1st 80 years in the 20th century the number of women earning Ph.D.'s was increasing at an increasing rate. (The positive changes in women/year was getting more and more positive those 80 years.) The concavity from 1980 to 2010 means the number of women earning Ph.D.'s was increasing at a decreasing rate. (Women are earning more Ph.D.'s but the rate of increase in women/year is decreasing.)

c. $\dfrac{1000-800}{1960-1950} = \dfrac{200}{10} = 20$ Ph.D.s/year;
$\dfrac{4000-1000}{1970-1960} = \dfrac{3000}{10} = 300$ Ph.D.s/year;
$\dfrac{9500-4000}{1980-1970} = \dfrac{4500}{10} = 450$ Ph.D.s/year

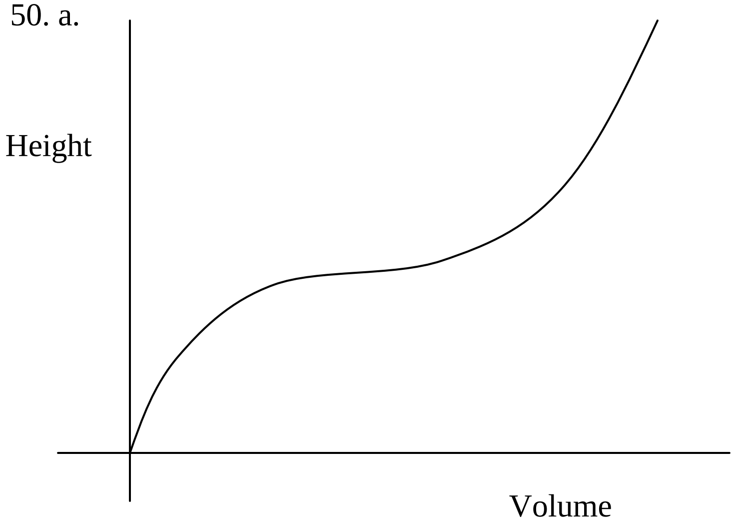
47.



48.



50. a.



b. The initial part of the graph is concave down which means that the height of the water in the container increases quickly at first as the volume increases but then increases slowly. When the concavity changes to concave up the height of the water in the container increases slowly at first but then more and more quickly.

4.2

1. If the 2^{nd} consecutive difference is constant.

2. “c” would be the distance the car was from Orlando after 0 hours of travel (initial position), “b” would be the rate at which the car was traveling in miles per hour initially (at time 0), “c” would represent the increase (if “a” is positive”) or decrease (if “a” is negative) in the change in the distance over each hour.

4. The change in the rate of change.

5. If the second difference is negative the graph will be concave down and if the second difference is positive the graph will be concave up.

6. The change in the rate of change is negative/decreasing.

7. The change in the rate of change is positive/increasing.

8. We know that c = 3 and a is a positive number.

11. Quadratic because the second difference is constant.

x	y	Δy	$\Delta(\Delta y)$
0	1		
1	3	2	
2	9	6	4
3	19	10	4
4	33	14	4

12. Quadratic because the second difference is constant.

x	y	Δy	$\Delta(\Delta y)$
2	-4		
4	-16	-12	-8
6	-36	-20	-8
8	-64	-28	-8
10	-100	-36	-8

13. Linear because the first difference is constant.

x	y	Δy	$\Delta(\Delta y)$
5	2		
10	4	2	
15	6	2	0
20	8	2	0
25	10	2	0

14. Neither because the first and second differences are not constant.

x	y	Δy	$\Delta(\Delta y)$
4	5		
8	10	5	
12	20	10	5
16	40	20	10
20	80	40	20

15. Quadratic because the second difference is constant.

x	y	Δy	$\Delta(\Delta y)$
-3	-1		
0	-10	-9	
3	-1	9	18
6	26	27	18
9	71	45	18

16. We are talking about what the second coordinate is when the first coordinate is 0. Initial often refers to when time is 0.

17. We are talking about the rate of change at the initial value.

18. If $a < 0$ then the rate of change is decreasing.

19. If $a > 0$ then the rate of change is increasing.

20. *a* cannot equal 0 because the a term is what determines the constant change in the rate of change. If $a = 0$ then the function would be linear and have a constant rate of change.

21. The parameter "*a*" is -0.00472 and represents the rate at which the million enrollees per year is changing by is -0.00944 million Medicare enrollees per year. The parameter "*b*" is 0.663 and represents the increase in the number of Medicare enrollees per year in the initial year of 1980. The parameter "*c*" is 28.4 and represents that the number of Medicare enrollees in 1980 is 28,400,000.

22. The parameter "*a*" is -2.889 and represents the rate at which the amount of money spent on prescription drugs per year is decreasing is 5.778 dollars per year. The parameter "*b*" is -2.613 and represents the decrease in the amount of money spent on prescription drugs per year in the initial year of 1990. The parameter "*c*" is 28.4 and represents the amount of money spent on prescription drugs in 1990.

23. The parameter "*a*" is 1.046 and represents the rate at which the number of children under 5 years-old per year is changing by is 2.092 children per year. The parameter "*b*" is 60.82 and represents the rate at which the number of children under 5 years-old is changing by is 60.82 per year. The parameter "*c*" is 2152 and represents the number of children under 5 years-old in 1990.

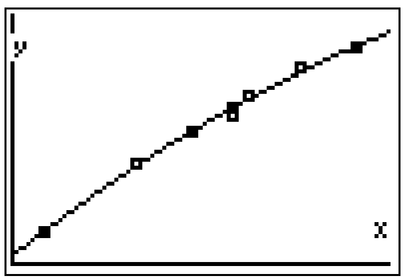
24. The parameter "*a*" is -140,281 and represents the rate at which the number of clinical malaria cases per year is decreasing is -280,562 cases per year. The parameter "*b*" is 658,186 and represents the increase in the number of clinical malaria cases per year in the initial year of 1998. The parameter "*c*" is 583,452 and represents the number of malaria cases in 1998.

26. The parameter "*a*" is -19.56 and represents the rate at which the population of the United Stats per year is decreasing -39.12 thousand people per year. The parameter "*b*" is 3407 and represents the increase in the number of thousand people in the United States per year in the initial year of 1990. The parameter "*c*" is 250,100 and represents the number of thousand people in the United States in 1990.

27. The parameter “*a*” is 141.25 and represents the rate at which the number of students enrolled in the Arizona Virtual Academy per year is increasing 282.5 students per year. The parameter “*b*” is 358.75 and represents the increase in the number of students enrolled in the Arizona Virtual Academy per year in the initial year of 2003. The parameter “*c*” is 318.75 and represents the number of students enrolled in the Arizona Virtual Academy in the year 2003.

30. The parameter “*a*” is 14.99 and represents the rate at which the amount of yogurt produced in millions of pounds per year is increasing 29.98 million pounds per year. The parameter “*b*” is 62.14 and represents the increase in yogurt production in millions of pounds per year in the initial year of 1997. The parameter “*c*” is 1555 and represents the millions of pounds of yogurt produced in 1997.

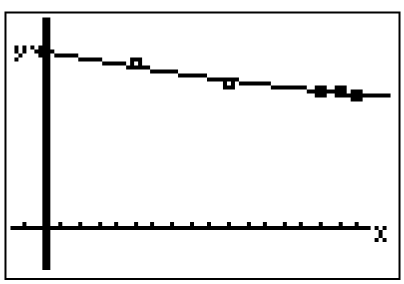
46.



$w(b) = -0.0009b^2 + 3.45b - 2101.29$

$a(20) \approx 308$ abortions in 2005.

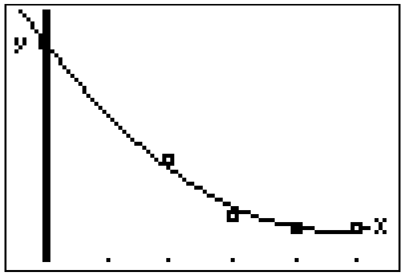
47.



$a(x) = 0.127x^2 - 8.339x + 423.504$

$a(1500) \approx 1006$ live white births when there are 1500 live births.

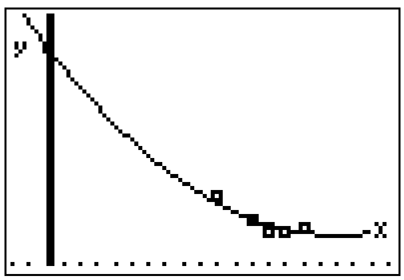
48.



$e(x) = 23.19x^2 - 218.07x + 1825.11$

$e(9) \approx 1741$ computer and electronic products industry employees in 2009.

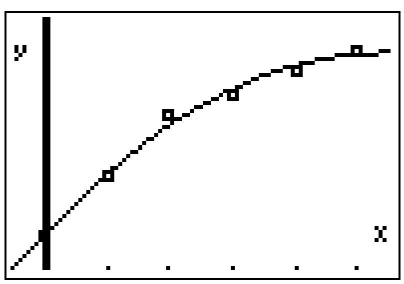
49.



$e(x) = 1.396x^2 - 47.62x + 841.912$

$e(19) \approx 441$ aerospace products and parts industry employees in 2009.

50.



$s(x) = -20.32x^2 + 221.01x + 790.93$

$s(8) \approx 1258.41$ thousand dollars is the NFL player's average salary in 2008.

56. For a function to be quadratic there has to be a constant change in the rate of change (constant second difference). For the data points (0,4), (3,13), and (10,34) there is a constant rate of change so these points form a ***linear*** function.

57.

x	y	$\frac{\Delta y}{\Delta x}$
0	4	
3	13	$\frac{13-4}{3-0}=3$
10	34	$\frac{34-13}{10-3}=3$

There is a constant rate of change between the data points therefore the function is linear and not quadratic.

58. No, because the constant second difference has to be determined when the increments between the x values are all the same.

x	y	$\frac{\Delta y}{\Delta x}$	$\Delta(\frac{\Delta y}{\Delta x})$
0	-1		
2	9	$\frac{9--1}{2-0}=5$	
4	27	$\frac{27-9}{4-2}=9$	4
8	53	$\frac{53-27}{8-4}=6.5$	-2.5
12	87	$\frac{87-53}{12-8}=8.5$	2